Sponsored Search Auctions

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Introduction

- Web search engines like Google and Yahoo! **monetize** their service **by auctioning** off advertising **space next to their standard algorithmic search results**.

Introduction

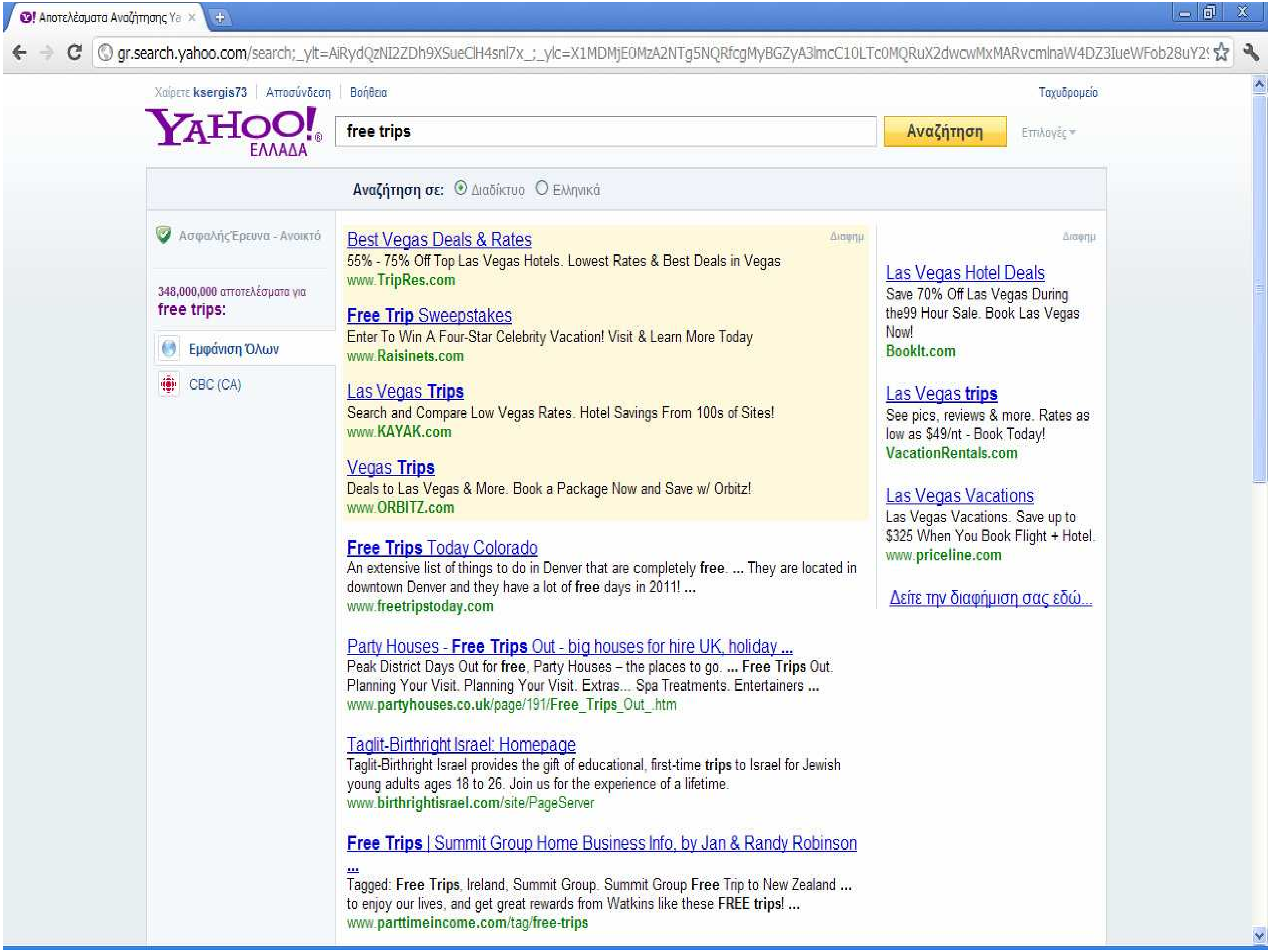
- For example, Apple or Best Buy may bid to appear among the advertisements – usually **located above** or **to the right** of the algorithmic results

Introduction

- These sponsored results are displayed in a format similar to algorithmic results:
  - as a list of items each containing
    - title,
    - text description
    - hyperlink to the advertiser’s Web page.

Introduction

- We call each position in the list a ***slot***.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Introduction

- **More than 50%** of **Web users** visit a **search engine every day**
- Americans conduct roughly **6 billion Web searches per month**
- **Over 13% of traffic** to commercial sites is **generated by search engines**
- **Over 40% of product searches** on the Web are **initiated via search engines**.

Introduction

- Today, Internet giants **Google** and **Yahoo!** boast a combined **market capitalization** of **over $150 billion**, largely on the strength of sponsored search.
- Roughly **85%** of **Google's $4.1 billion** and roughly **45%** of **Yahoo!'s $3.7 billion** in **2005 revenue** is likely attributable to sponsored search.

Introduction

- Advertisers specify:
  - List of pairs of keywords
  - Bids
  - Total maximum daily or weekly budget.
- Every time a user searches for a keyword, an auction takes place among the set of interested advertisers who have not exhausted their budgets.

Existing Models

- Static
  - Vickrey Clarke Grooves Mechanism (VCG)
  - Generalized First Price (GFP)
  - Generalized Second Price (GSP)
- Dynamic
  - On-line Allocation Problem

Static

- n bidders/advertisers
- k slots (k is fixed apriori – $k<n$)
- α_{ij} as a click through rate (CTR) of the bidder j if placed in slot i
- v_j is the value of the bidder j for a click

Static

- Assumptions
  - Bidders prefer a higher slot to a lower slot

$$\alpha_{ij} \geq \alpha_{i+1,j} \text{ for i=1,2,...,} k-1$$

  - v_i is independent of the slot position (**static**)
  - CTR for a slot does not depend on the identity of other bidders.
  - CTRs are assumed to be common knowledge (**static** nature)
    - not the reality - CTRs can fluctuate dramatically over small periods)

Static

- Revenue Maximization
- Allocative Efficiency

Revenue Maximization

- Result of Myerson
- The generalized Vickrey auction is applied not to the actual values v_j but to the corresponding virtual values
- Generalized Vickrey auction with reserve prices

Revenue Maximization

- Maximization bidder payments:

$$\max \sum_{j=1}^{n} p_j$$

Revenue Maximization

- Surplus Allocation:

$$\max \sum_{j=1}^{n} x_j(b) v_j$$

$x_j(b)$: expected CTR of bidder j who bids b

- Virtual Surplus Allocation:

$$\max \sum_{j=1}^{n} x_j(b) \varphi_j(v_j)$$

v_j : drawn ind/ntly from continuous prob. distribution

  - where:

$$\varphi_j(v_j) = v_j - \frac{1 - F_j(v_j)}{f_j(v_j)}$$

$$F_j(z) = \Pr\left[v_j \le z\right] \quad , \quad f_j(z) = \frac{d}{dz} F_j(z)$$

Revenue Maximization

- Expected **Profit** of a Truthful Mechanism **M**, is equal to the Expected Virtual Surplus:

$$E_t(M(t)) = E_t\left[\sum_j \varphi_j(v_j)x_j(t)\right]$$

  - Proof:

$$E_b(p_j(b)) = \int_{b=0}^{h} p_j(b)f(b)db = \ldots = E\left[\varphi_j(b)x_j(b)\right]$$

    - Mechanism Truthful in Expectation:
      - $x_j(b)$ Monotone non-decreasing
      - $p_j(b) = b_j x_j(b) - \int_0^b x_j(z)dz$

Revenue Maximization

- Thus, Virtual surplus is truthful

  if and only if

  $\varphi_j(v_j)$ is monotone non-decreasing in v_j

- Myerson Mechanism:
  - Given bids b and F (here Bayesian – Nash distribution), compute 'virtual bids': $b'_i = \varphi_i(b_i)$
  - Run VCG on b' to get x' and p'
  - Output x=x' and p with $p_i = \varphi_i^{-1}(p_i')$

Revenue Maximization

- F is the Bayesian – Nash distribution of of the generalized Vickrey (second price) auction (second price) with reserve prices
- Proof similar with the Vickrey (second price) auction (second price) with reserve price for 1 item

Revenue Maximization

- Revenue without reserve price:

$$R_0 = \frac{1}{3}$$

- Revenue with reserve price r:

$$r = \frac{1}{2} \quad , \quad R_{1/2} = \frac{5}{12}$$

Revenue Maximization

- Revenue without reserve price:
  - Given V_A, B's valuation is likely to lie anywhere between 0 and V_A
  - On average $V_B = V_A/2$
  - On average, V_B halfway between 0 and V_A
  - On average, V_A halfway between V_B and 1

Revenue Maximization

- Revenue without reserve price:
  - $E[V_B] = 1/3$ and $E[V_A] = 2/3$
  - $E[V_B] = E[V_A]/2 = 1/3$

Revenue Maximization

- Revenue with reserve price r:
  - It may be the case that a bidder has positive valuation but negative virtual valuation.
  - Thus, for allocating a single item, the optimal mechanism finds the bidder with the largest nonnegative virtual valuation if there is one, and allocates to that bidder

Revenue Maximization

- Revenue with reserve price r:
  - bidder 1 (same for bidder 2) wins precisely when:

$$\varphi_1(b_1) \geq \max\{\varphi_2(b_2), 0\} \quad \Rightarrow$$

$$p_1 = \inf\{b : \varphi_1(b) \geq \varphi_2(b_2) \wedge \varphi_1(b) \geq 0\}$$

  - Since $\varphi_1 = \varphi_2 = \varphi$

$$p_1 = \min\{b_1, \varphi^{-1}(0)\} = \varphi^{-1}(0)$$

  - For

$$F(z) = z \ , \ f(z) = 1 \ \Rightarrow \ \varphi(z) = 2z - 1 \ \Rightarrow \ \varphi^{-1}(0) = \frac{1}{2}$$

Revenue Maximization

- Revenue with reserve price r:
  - For r=1/2:
    - Pr[both below 1/2]=1/2*1/2=1/4
    - Pr[both above 1/2]=1/2*1/2=1/4
    - Pr[one above 1/2]=1/2
    - Est. payoff both below = 0
    - Est. payoff both above = 4/6
    - Est. payoff one above = 1/2

$$R_{1/2} = \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot \frac{4}{6} + \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{12}$$

Allocative Efficiency

- Let $x_{ij} = 1$ if bidder j is assigned slot i
- $x_{ij} = 0$ otherwise

VCG

- Solution of LP:

$$\max \sum_{i=1}^{k} \sum_{j=1}^{n} \alpha_{ij} v_j x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} \leq 1 \quad , \quad \forall i=1,2,\ldots,k$$

$$\sum_{i=1}^{k} x_{ij} \leq 1 \quad , \quad \forall j=1,2,\ldots,n$$

$$x_{ij} \geq 0 \quad , \quad \forall i=1,2,\ldots,k \quad , \quad \forall j=1,2,\ldots,n$$

VCG

- Dual:

$$\min \sum_{i=1}^{k} p_i + \sum_{j=1}^{n} q_j$$

$$\text{s.t.} \quad p_i + q_j \geq \alpha_{ij} v_j \quad , \quad \forall i=1,2,\ldots,k \quad , \quad \forall j=1,2,\ldots,n$$

$$p_i, q_j \geq 0 \quad , \quad \forall i=1,2,\ldots,k \quad , \quad \forall j=1,2,\ldots,n$$

p_i : expected payment bidder

q_j : expected profit bidder

VCG

- Special Case:
  - CTRs bidder independent:

    $\alpha_{ij} = \mu_i$
  - Simple algorithm Northwest Corner Rule:
    - Assign bidder with highest value top slot, second highest value second slot e.t.c
      - **Assortative** assignment

VCG

- Cons
  - requires solving a computational problem which needs to be done online for every search and is expensive
  - Other mechanisms better revenues than VCG

GFP

- Let b1,…,bn be the bids. The GFP mechanism is as follows:
  - Sorts bidders according to the bids b1,…,bn.
  - Assigns slots according to the order (assign top slot to the highest bidder and so on).
  - Charge bidder i according to his bid.
- Yahoo! used a GFP auction until 2004.

GSP

- Let w1,…,wn be the weights on bidders which are static and independent of the bids b1,…,bn. The GSP mechanism is as follows:
  - Sort bidders by $s_i = w_i b_i$
    - (assume $s_1 \geq s_2 \geq \ldots \geq s_n$)
  - Allocate slots to bidders 1 ,…,k in that order (i.e., bidder i gets the ith slot if $i \leq k$).
  - Charge i the mininum bid he needs to retain his slot (i.e., $p_i = \frac{s_{i+1}}{w_i}$).

GSP

- Overture model: For every i, $w_i = 1$ (bidders ordered according to the bids only).
- Google model: Google assigns weights based on the CTR at the top slot $w_i \simeq \alpha_{i1}$. The assumption here is that α_{i1} is static (or slow changing)
- This ordering is also called ‘revenue order’ since $s_i = \alpha_{i1} b_i$ is the expected revenue if i is put in slot 1 and there is only one slot.

GFP not truthful

- Payoff in general: $c_{ij}(v_j - p_j)$

Table 1: GFP example

Advertiser	v_i	b_i	Slot	c_i	$p_i c_i$	Total payoff
Alice	50	40	1	10	400	100
Bob	20	19	2	5	95	5
Charlie	2	2	None	0	0	0

GSP not truthful

- Payoff in general: $c_{ij}(v_j - p_j)$

Table 2: GSP example

Advertiser	v_i	b_i	Slot	c_i	$p_i c_i$	Total payoff
Alice	50	40	1	10	190	310
Bob	40	19	2	5	10	105
Charlie	2	2	None	0	0	0

GSP not truthful

- Payoff in general: $c_{ij}(v_j - p_j)$

Table 3: GSP example - true bids

Advertiser	v_i	b_i	Slot	c_i	$p_i c_i$	Total payoff
Alice	50	50	1	10	400	100
Bob	40	40	2	5	10	190
Charlie	2	2	None	0	0	0

GSP not truthful

- Payoff in general: $c_{ij}(v_j - p_j)$

Table 4: GSP example - Alice's strategy

Advertiser	v_i	b_i	Slot	c_i	$p_i c_i$	Total payoff
Alice	50	3	2	5	10	240
Bob	40	40	1	10	30	370
Charlie	2	2	None	0	0	0

VCG Payoff

- Payoff in general: $c_{ij}(v_j - p_j)$

Table 5: VCG payoffs

Advertiser	v_i	b_i	Slot	c_i	$p_i c_i$	Total payoff
Alice	50	50	1	10	210	290
Bob	40	40	2	5	10	190
Charlie	2	2	None	0	0	0

- eachbidder j would be made to pay the sum of

$$(c_{i-1} - c_i)b_i$$

for every I below him

GSP vs VCG

- Search engines revenues under GSP better than VCG:

$$c_i p_i^{VCG} - c_{i+1} p_{i+1}^{VCG} = (c_i - c_{i+1}) b_{i+1} \leq c_i b_{i+1} - c_{i+1} b_{i+2} = c_i p_i - c_{i+1} p_{i+1}$$

Equilibrium Properties

- GFP: Bayes-Nash symmetric equilibrium
  - argument identical to that of the sealed bid **first price auction** for a single good for symmetric bidders (*same distributions*) the revenue equivalence theorem implies that revenue from GFP is the same as any other auction that allocates according to bid order.
  - **Revenue Equivalence Principle** *Under certain weak assumptions, for every two Bayesian–Nash implementations of the same social choice function* f *, we have that if for some type* t′ *of player* i*, the expected (over the types of the other players) payment of player* i *is the same in the two mechanisms, then it is the same for every value of* i*'s type* t*.*

Equilibrium Properties

- GSP: Today nothing is known about the Bayesian equilibrium of the GSP auction
- Special Case:
  - CTRs are separable:

$$\alpha_{ij} = \mu_i \beta_j$$

$$\text{special case:}$$

$$\alpha_{ij} = \mu_i$$

- Locally Envy-Free equilibria

GSP Equilibrium Properties

- Retaliation:

  Suppose advertiser k bids b_k $\rightarrow$ assigned to position i, and advertiser k' bids $b_{k'} > b_k$ $\rightarrow$ assigned to position (i - 1).

  If k raises his bid slightly, his own payoff does not change, but the payoff of the player above him decreases

  k' can retaliate...

GSP Equilibrium Properties

- Vector of bids changes all time
- What if the vector converges to a **rest point**?
- An advertiser in position i should not want to "exchange" positions with the advertiser in position (i-1)
- "**locally envy-free**" vectors

GSP Equilibrium Properties

- *An equilibrium of the simultaneous-move game (Γ) induced by GSP is locally envy-free if a player cannot improve his payoff by exchanging bids with the player ranked one position above him*

$$\mu_i v_{g(i)} - p_i \geq \mu_{i-1} v_{g(i)} - p_{i-1}$$

GSP Equilibrium Properties

- LEMMA 1: *The outcome of any locally envy-free equilibrium of auction Γ is a stable assignment.*
- *Proof:*
  - no advertiser can profitably rematch with a position assigned to an advertiser below him (equilibrium)

$$\mu_i v_{g(i)} - p_i \geq \mu_{i+1} v_{g(i)} - p_{i+1}$$

GSP Equilibrium Properties

- *Proof (cont):*
  - show that no advertiser can profitably rematch with the position assigned to an advertiser more than one spot above him
  - locally envyfree equilibrium: matching must be **assortative**

$$\mu_i v_{g(i)} - p_i \geq \mu_{i+1} v_{g(i)} - p_{i+1}$$

$$\mu_{i+1} v_{g(i+1)} - p_{i+1} \geq \mu_i v_{g(i+1)} - p_i$$

$$\text{thus:}$$

$$(\mu_i - \mu_{i+1}) v_{g(i)} \geq (\mu_i - \mu_{i+1}) v_{g(i+1)}$$

GSP Equilibrium Properties

- *Proof (cont):*

Suppose $m \leq i$:

$$\mu_i v_{g(i)} - p_i \geq \mu_{i-1} v_{g(i)} - p_{i-1}$$

$$\mu_{i-1} v_{g(i-1)} - p_{i-1} \geq \mu_{i-2} v_{g(i-1)} - p_{i-2}$$

.

.

.

$$\mu_{m+1} v_{g(m+1)} - p_{m+1} \geq \mu_m v_{g(m+1)} - p_m$$

thus :

$$\mu_i v_{g(i)} - p_i \geq \mu_m v_{g(i)} - p_m$$

GSP Equilibrium Properties

- LEMMA 2: *If the number of advertisers is greater than the number of available positions then any stable assignment is an outcome of a locally envy-free equilibrium of auction Γ*
- *Proof:*
  - stable assignment ⇒ assortative ⇒ advertisers are labeled in decreasing order of their bids:

$$v_j > v_k \Leftrightarrow j < k$$

  - Thus, advertiser i match with position i, payment i

GSP Equilibrium Properties

- *Proof (cont):*
  - Let:

$$b_1 = v_1$$

and

$$b_i = \frac{p_{i-1}}{\mu_{i-1}} \quad \text{for} \quad i > 1$$

GSP Equilibrium Properties

- *Proof (cont):*
  - Let:

$$b_i > b_{i+1}$$

otherwise:

$$\frac{p_{i-1}}{\mu_{i-1}} \le \frac{p_i}{\mu_i} \Rightarrow v_i - \frac{p_{i-1}}{\mu_{i-1}} \ge v_i - \frac{p_i}{\mu_i} \Rightarrow \mu_{i-1} v_i - p_{i-1} \ge \mu_i v_i - p_i$$

  - So, deviating and moving to a different position in this strategy profile is at most as profitable for any player as rematching with the corresponding position in the assignment game Γ

GSP Equilibrium Properties

- Let assign:

$$p_i \rightarrow p_i^{VCG}$$

  - THEOREM 1: *Strategy profile B* is a locally envy-free equilibrium of game Γ. In this equilibrium, each advertiser's position and payment are equal to those in the dominant-strategy equilibrium of the game induced by VCG. In any other locally envy-free equilibrium of game Γ, the total revenue of the seller is at least as high as in B*.*

GSP Equilibrium Properties

- Proof:
  - Payments under strategy profile B^* coincide with VCG $\Rightarrow$ B^* locally envy-free equilibrium (construction)
  - This assignment is:
    - **Best** stable assignment for all **advertisers**
    - **Worst** stable assignment for **auctioneers**

GSP Equilibrium Properties

- In any stable assignment:

$$p_k \geq \mu_{k+1} v_k = p_k^{VCG}$$

otherwise advertiser k+1 would find it profitable to match with position k. Next,

$$p_{k-1} - p_k \geq (\mu_{k-1} - \mu_k) v_k$$

otherwise advertiser k would find it profitable to match with position k-1

$$p_{k-1} - p_k \geq (\mu_{k-1} - \mu_k) v_k \Rightarrow$$

$$p_{k-1} \geq (\mu_{k-1} - \mu_k) v_k + p_k = (\mu_{k-1} - \mu_k) v_k + p_k^{VCG} \geq p_{k-1}^{VCG}$$

Dynamic Aspects

- Online Allocation Problem
  - Auctions are repeated with great frequency
  - Model them as repeated games of incomplete information
  - For simplicity we assume that each page has only one slot for advertisements.
  - The objective is to maximize total revenue while respecting the budget constraint of the bidders

Online Allocation Problem

- n number of advertisers and m the number of keywords.
- advertiser j has a bid of b_{ij} for keyword i and a total budget of B_j.
- Bids are small compared to budgets
- Since search engine has an accurate estimate of r_i, the number of people searching for keyword i for all $1 \le i \le m$, it is easy to approximate the optimal allocation using a simple LP
- x_{ij} be the total number of queries on keyword i allocated to bidder j

Online Allocation Problem

- LP:

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{m} \sum_{j=1}^{n} b_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \le r_i && \forall 1 \le i \le m \\
& \sum_{i=1}^{m} b_{ij} x_{ij} \le B_j && \forall 1 \le j \le n \\
& x_{ij} \ge 0 && \forall 1 \le i \le m, \quad \forall 1 \le j \le n
\end{aligned}
$$

Online Allocation Problem

- Dual:

$$
\begin{aligned}
\min \quad & \sum_{j=1}^{n} B_j \beta_j + \sum_{i=1}^{m} r_i \alpha_i \\
\text{s.t.} \quad & \alpha_i + b_{ij}\beta_j \geq b_{ij} \quad \forall 1 \leq i \leq m, \forall 1 \leq j \leq n \\
& \beta_j \geq 0 \quad \forall 1 \leq j \leq n \\
& \alpha_i \geq 0 \quad \forall 1 \leq i \leq m
\end{aligned}
$$

Online Allocation Problem

- Complementary slackness:

  $$b_{ij}(1-\beta_j)=\alpha'=\max b_{ik}(1-\beta_k)\ ,\ 1\leq k\leq n$$

- Search engine allocates its corresponding advertisement space to the bidder j with the highest $b_{ij}\ (1-\beta_j)$
- if we allocate keyword i to agent now we obtain an immediate 'payoff' of b_{ij}.
- However, this consumes b_{ij} of the budget $\Rightarrow$ opportunity cost of $b_{ij}\beta_j$.
- Reasonable to assign keyword i to j provided

  $$b_{ij}(1-\beta_j) > 0$$

Online Allocation Problem

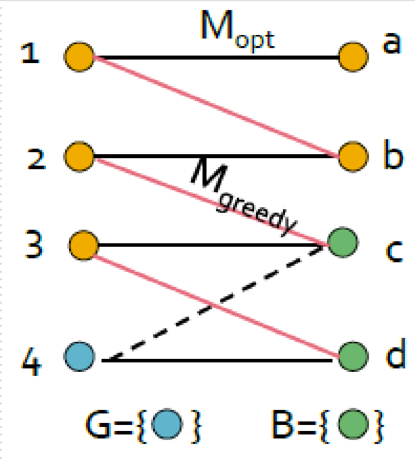
- Greedy:
  - among the bidders whose budgets are not exhausted, allocate the query to the one with the highest bid
- *competitive ratio*—the ratio between online algorithm’s performance and the optimal offline algorithm's performance
- Competitive ratio of greedy algorithm is 1/2

Online Allocation Problem

- Greedy procedure is not guaranteed to find the optimum solution:
  - 2 bidders each with a budget of \$2.
    - $b_{11} = 2$, $b_{12} = 2 - \varepsilon$, $b_{21} = 2$, $b_{22} = \varepsilon$
  - If query 1 arrives before query 2, it will be assigned to bidder 1.
  - bidder 1's budget is exhausted. When query 2 arrives, it is assigned to bidder 2.
  - Objective Function value of $2 + \varepsilon$.
  - The optimal solution would assign query 2 to bidder 1 and query 1 to bidder 2, yielding an objective function value of $4 - \varepsilon$.

Online Allocation Problem

- Similar to Graph Matching Problem:
  - Consider the set G of girls matched in Mopt but not in Mgreedy
  - Then every boy B adjacent to girls in G is already matched in Mgreedy: |B| ≤ |Mgreedy|
  - There are at least |G| such boys (|G| ≤ |B|) otherwise the optimal algorithm could, not have matched all the G girls. So: |G| ≤ |Mgreedy|
  - By definition of G also: |Mopt| ≤ |Mgreedy| + |G|
  - |Mgreedy|/|Mopt| ≥ 1/2



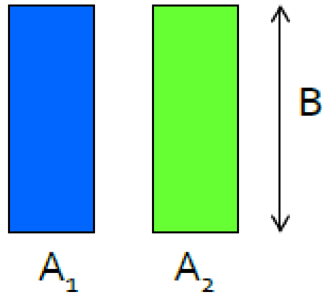
Online Allocation Problem

- Can we do better?
- BALANCE algorithm:
  - For each query, pick the advertiser with the largest unspent budget

Online Allocation Problem

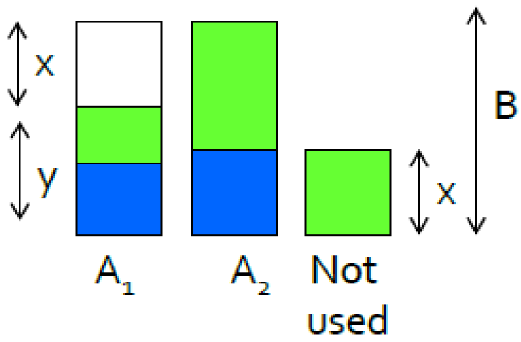
- Two advertisers A and B
- A bids on query x, B bids on x and y
  - Both have budgets of $4
- Query stream: xxxxyyyy
  - BALANCE choice: ABABBB__
  - Optimal: AAAABBBB
- Competitive ratio = ¾

Analyzing BALANCE



Queries allocated to A_1 in optimal solution

Queries allocated to A_2 in optimal solution



Opt revenue = 2B
Balance revenue = 2B-x = B+y

We have $y \geq x$
Balance revenue is minimum for x=y=B/2
Minimum Balance revenue = 3B/2
Competitive Ratio = 3/4

BALANCE: General Result

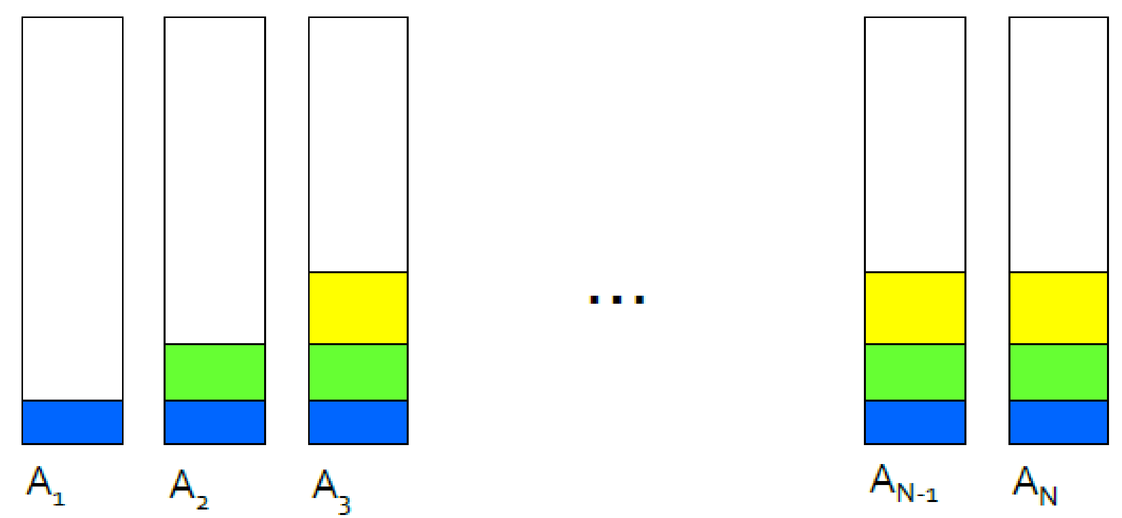
- In the general case, worst competitive ratio of BALANCE is
  - $1-1/e$ = approx. 0.63

- Let's see the worst case that gives this ratio

Worst Case for BALANCE

- N advertisers: A1, A2, … AN
  - Each with budget B > N
- Queries: N·B queries appear in N rounds of B queries each:
  - Bidding:Round 1 queries: bidders A1, A2, …, AN
  - Round 2 queries: bidders A2, A3, …, AN
  - Round queries: bidders Ai, …, AN
- Optimum allocation: Allocate round i queries to Ai

Worst Case for BALANCE



BALANCE Algorithm

- β_j's as a function of the bidders spent budget

$$\phi(x) = 1 - e^{x-1}$$

$$\beta_j = 1 - \phi(f_j)$$

- β_j's as a function of the bidders spent budget
- f_j: the fraction of the budget of bidder j , which has been spent
- **Algorithm:** Every time a query i arrives, allocate its advertisement space to the bidder j , who maximizes $b_{ij}\varphi(f_j)$

BALANCE Algorithm

- Let k be a sufficiently large number used for discretizing the budgets of the bidders.
- Advertiser is of type j if she has spent within (j−1/k , j/k] fraction of budget so far.
- s_j: Total budget of type j bidders.
- For i = 0, 1, . . . , k, define w_i: Amount of money spent by all bidders from the interval (i−1/k , i/k] of their budgets
- Discrete version of function φ:

$$\Phi(s) = 1 - \left(1 - \frac{1}{k}\right)^{k-s}$$

BALANCE Algorithm

- When k tends to infinity:

$$\Phi(s) \rightarrow \phi(\frac{s}{k})$$

- Let OPT be the solution of the optimal off-line algorithm

BALANCE Algorithm

- **Lemma:** *At the end of the algorithm, this inequality holds:*

$$\sum_{i=0}^{k} \Phi(i) s_i \leq \sum_{i=0}^{k} . \Phi(i) w_i$$

BALANCE Algorithm

- Lemma Proof:
  - Consider time query q arrives.
  - OPT allocates q to a bidder of current type t , whose type at the end of the algorithm will be t′.
  - b_{opt} , b_{alg}: amount of money that OPT and the BALANCE get from bidders for q.
  - Let i be the type of the bidder that the algorithm allocates the query

$$\Phi(t')b_{\text{opt}} \leq \Phi(t)b_{\text{opt}} \leq \Phi(i)b_{\text{alg}}$$

BALANCE Algorithm

- **Theorem:** *The competitive ratio of Algorithm 1 is* 1 − 1/e.
- Proof:
  - By definition: $w_i \leq \frac{1}{k} \sum_{j=i}^{k} s_j$

- Thus:

$$\sum_{i=0}^{k} \Phi(i) s_i \leq \frac{1}{k} \sum_{i=0}^{k} \Phi(i) \sum_{j=i}^{k} s_j$$

- We conclude that:

$$\left(\Phi(0) - o\left(\frac{1}{k}\right)\right) \sum_{i=0}^{k} s_i \leq \sum_{i=0}^{k} \frac{i}{k} s_i$$

- Note that as k goes to infinity the left-hand side tends to (1 − 1/e)OPT. Right-hand revenue of the BALANCE

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